Body Wave Velocity Dispersion in Layered Periodic Media
I.O. BAYUK*, United Institute of Physics of the Earth, Russian Academy of Sciences, Bol’shaya Gruzinskaya ul., 10, Moscow, 123810 Russia
E.M. CHESNOKOV, United Institute of Physics of the Earth, Russian Academy of Sciences, Bol’shaya Gruzinskaya ul., 10, Moscow, 123810 Russia

Summary

Medium consisting of two periodically alternating isotropic layers is considered. The direct body waves propagating at different frequencies along and normal to the layers are studied. Each of these waves is characterized by wave number(s) \( k \) following from solution to dispersion equation constructed for studied wave. It is shown that, commonly, solutions to the dispersion equations have many branches giving roots for the wave number. For waves propagating in the layers, the branches are continuous curves relative to frequencies. Curves obtained for P-wave propagating normal to the layers are periodic and exhibit gaps (forbidden zones). The facts that the group wave velocities must be positive and displacements are real functions are taken as conditions for the root selection. At limiting case of low frequencies, the solutions to dispersion curves are unique for each wave and characterize the effective properties of medium having hexagonal elastic symmetry. In this case, the solution is compared to that obtained in correlation approximation for thin-layered medium. Media containing layers of different materials are considered.

Introduction

Rytov (1956) has derived dispersion equations for medium with alternating isotropic layers in order to obtain solution for limiting case for low frequency, or, for thin-layered media. The solutions to the equations have not been obtained for arbitrary frequency. However, to study wave propagation in wide frequency range for this type of media is of interest for geophysics, since layering is commonly observed in sedimentary rocks.

Theory

In this work, the Rytov’s dispersion equations are solved in the case of arbitrary frequency. The considered waves are as follows: (1) waves having simultaneously longitudinal and transverse components and propagating along layers (horizontal direction), (2) pure shear wave propagating along the layers, and (3) waves propagating directly along vertical axis. For all waves, the amplitude of the displacement components are periodic functions of \( z \) (xy is the layer plane). In the first case, two type of waves are studied - the waves that are longitudinal and shear “in average” (over the period). The dispersion equations for these waves are determinants of 4th-rank matrix following from boundary conditions specified for each wave on interfaces. Solutions to these equations are numerically obtained as functions \( \omega(k) \) and \( \omega(V) \) (\( V \) is velocity). The solution to each equation is non-unique and represented as continuous functions of frequency. Figure 1 represent wave-number branches for S-wave velocity propagating along \( x \)-direction and having displacement components along \( x \) and \( z \) directions within interval of wave numbers corresponding to velocities in constituting layers. The considered medium consists of 100-m clay and 50-quartz layers, periodically alternating.

The only case, when the dispersion equation can be solved analytically for the wave vector is P-wave propagating along \( z \) axis.

Figure 2 shows the function \( \omega(k) \) for P-wave propagating along \( z \) axis for the medium presented in Fig. 1. Straight lines 1 and 2 give wave numbers for layers. We can see that only one curve exists within the straight lines (black dashed curve). This curve has gaps (forbidden zones) for some frequency ranges. The tangent of this curve is group wave velocity in this direction (constant value at all frequencies).

Conclusions

The solutions of dispersion equations for direct body waves propagating in different directions of periodically layered medium are obtained. In general, the solution has a form of branches that are continuous for propagation in horizontal plane (plane of layers) and discrete for P-wave in the vertical direction.

References


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Fig. 1. Wave numbers for shear wave velocity propagating in the \( xy \) plane

Fig. 2. Wave number for longitudinal wave velocity propagating normal to layers